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THE PROCESS OF MAGNETIC FLUX PENETRATION INTO SUPERCONDUCTORS

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Abstract

In the present paper the magnetic flux penetration dynamics of type-II superconductors in the flux creep regime is studied by analytically solving the nonlinear diffusion equation for the magnetic flux induction, assuming that an applied field parallel to the surface of the sample and using a power-law dependence of the differential resistivity on the magnetic field induction. An exact solution of nonlinear diffusion equation for the magnetic induction B(r, t) is obtained by using a well known self-similar technique. We study the problem in the framework of a macroscopic approach, in which all lengths scales are larger than the flux-line spacing; thus, the superconductor is considered as an uniform medium.

Key words: *magnetic flux penetration, critical state, nonlinear diffusion*

1. Introduction

Theoretical investigations of the magnetic flux penetration dynamics into superconductors in a various regimes with a various current-voltage characteristics is one of key problems of electrodynamics of superconductors. Mathematical problem of theoretical study the dynamics of evolution and penetration of magnetic flux into the sample in the viscous flux flow regime can be formulated on the basis of a nonlinear diffusion-like equation [1, 2] for the magnetic field induction in a superconductor [3-5]. The dynamics of space-time evolution of the magnetic flux penetration into type-II superconductors, where the flux lines are parallel to the surface of the sample for the viscous flux flow regime with a nonlinear relationship between the field and current density in type II superconductors has been studied by many authors [4-14]. The magnetic flux penetration problem was theoretically studied for the particular case, where the flux flow resistivity independent of the magnetic field by authors [5]. Similar problem has been considered in [6] for the semi-infinite sample in parallel geometry. The situation, where flux flow resistivity depends linearly on the magnetic field induction was considered analytically in [4]. Analogical problem for the creep regime with a nonlinear relationship between the current and field has been considered in [8-14]. The magnetic flux penetration into the superconductor sample, where the flux lines are perpendicular to the surface of the sample is described by a non-local nonlinear diffusion equation [7]. This problem has been exactly solved by Briksin and Dorogovstev [7] for the case thin film geometry in the flux flow regime of a type-II superconductors. In the present paper the magnetic flux penetration dynamics of type-II superconductors in the flux flow regime is studied by analytically solving the nonlinear diffusion equation for the magnetic flux induction, assuming that an applied field parallel to the surface of the sample and using a power-law dependence of the differential resistivity on the magnetic field induction. An exact solution of nonlinear diffusion equation for the magnetic induction $\vec{B}(r,t)$ is obtained by using a well known self-similar technique. We study the problem in the framework of a macroscopic approach, in which all lengths scales are larger than the flux-line spacing; thus, the superconductor is considered as an uniform medium.

2. Formulation of the problem

Bean [15] has proposed the critical state model which is successfully used to describe magnetic properties of type II superconductors. According to this model, the distribution of the magnetic flux density $\vec{j}$ and the transport current density inside a superconductor is given by a solution of the equation

$$rot\vec{B} = \frac{4\pi}{c}\vec{j} \tag{1}$$

When the penetrated magnetic flux changes with time, an electric field $\vec{E}(r,t)$ is generated inside the sample according to Faraday's law

$$rot\vec{E} = -\frac{1}{c}\frac{d\vec{B}}{dt} \tag{2}$$

The motion of vortex filaments with a velocity v leads to the appearance of an electric field

$$\vec{E} = \frac{v}{c}\vec{B} \tag{3}$$

In general, the dynamics of magnetic flux penetration process is determined by many external and internal factors, as by the sweep rate of the external magnetic field, the type of voltage-current characteristics, the critical current density and its magnetic field and temperature derivatives, the profile temperature and surface cooling conditions, the sample geometry and pinning properties of the considered sample. Many results on this problem, have been obtained in the flux flow regime, where voltage current-current characteristics of hard superconductor is described by linear dependence of $j(E)$ at sufficiently large values of electric field [7]. The results for the flux flow state the value of B_j depends mainly on the critical current density and the specific heat of the sample. The nonlinear part of the curve $j(E)$ in the region of weak electric fields is associated with flux creep. In the flux creep regime the magnitude of flux penetration profile strongly depends on variation of external parameters, in particular, on the magnetic field sweep rate [10, 11]. In this paper we shall discuss the effect of flux creep and the nonlinear current-voltage characteristics on the profile of the magnetic flux penetration process, qualitatively. The current-voltage characteristics of type- II conventional as well as high $-T_c$ superconductors in the flux creep regime is a highly nonlinear due to thermally activated magnetic flux motion. Thermally activated flux motion or flux creep problem in superconductor samples with various geometries and conditions has been recently extensively studied by authors [7, 14].

For the simple case it can be presented by Kim-Anderson [16, 17] formulae

$$U(\vec{j}) = U_0(1 - \frac{\vec{j}}{j_c}) \tag{4}$$

here U_0 is the characteristic scale of the activation energy and $j_c = j_c(B)$ is the critical current density. In the flux creep state the effective activation energy U grows logarithmically [18, 19] with decreasing current density as

$$U(\vec{j}) = U_0(\frac{\vec{j}}{j_c})^n \tag{5}$$

where the exponent n depends upon the flux creep regime. The expression (43) gives a quite realistic description for activation barriers in a wide range of temperatures and magnetic fields. The power law characteristic dependence for $U(\vec{j}$ has been observed in numerous experiments, and it has been used extensively in recent theoretical studies for the field and current distributions in superconductors [19]. Equation (5) is equivalent to a power law for the flux-flow resistivity

$$v = v_0|\frac{j}{j_c}|^n \tag{6}$$

Here v0 is the velocity of the thermally activated flux motion at zero temperature at $T = 0, U$ is the activation energy due to vortex pinning. The activation energy $U(\vec{j}) = U(j, \vec{BT})$ depends on temperature T, magnetic field induction B and current density j. In this case the phenomenological relation $E(j)$ may be chosen in the power-law form

$$\vec{E} = v_0|B||\frac{\vec{j}}{j_c}|^n\vec{j} \tag{7}$$

If n = 1 the last equation reduces to Ohm's law, describing the normal or flux-flow state. For infinitely large n, the equation describes the Bean critical state model $j_c = j_c(B_e)$. When $1 < n < \infty$, the last equation describes nonlinear flux creep. In such case the analytic solution of the nonlinear creep equation can be constructed by choosing the critical current dependence on the magnetic field.

The dependence of U on the current density j is extensively discussed in the literature [8]. In particular, the authors of [7] have analyzed the flux creep at different dependencies of the activation energy U on field B and current density j. For the vortex glass and collective creep models the potential barriers highly nonlinear function of j [18]. A voltage-current characteristic of a type-II superconductor in the flux creep state is characterized by the power or exponential law increase of E with increasing j. For the linear current dependence of the potential barrier $U(j)$ [19], the dependence $j(E)$ has the form

$$\vec{j} = j_c + j_1 ln|\frac{\vec{E}}{E_c}| \quad (8)$$

where parameter j1 determines the slope of the j-E curve and it is assumed $j_1 << j_c$. In this case the differential conductivity σ is determined by the following expression (see, Refs. [10, 11])

$$\sigma = \frac{dj}{dE} = \frac{j_1}{E}| \quad (9)$$

For the logarithmic current dependence of the potential barrier $U(j)$ proposed by Zeldov et.all. [18]

$$U(\vec{j}) = U_0 ln|\frac{j_c}{\vec{j}}|^n \quad (10)$$

the dependence $j(E)$ has the form

$$\vec{j} = j_c ln|\frac{\vec{E}}{E_c}|^{1/n} \quad (11)$$

when, the flux creep is determined by numerous spatial defects of the sample. $U_0 = const$ and E_c is the crossover electric field. Here the parameter $n = U_0/kT$ is a function of temperature T, magnetic field H and depends on the pinning regimes and can vary widely for various types of superconductors. In the case $n = 1$ the power-law relation (33) reduces to Ohm's law, describing the normal or flux-flow regime. For infinitely large n, the equation describes the Bean critical state model $j_c(B_e)$. When $1 < n < 1$, this equation describes nonlinear flux creep [18]. In this case the differential conductivity σ is determined by the following expression

$$\sigma = \frac{dj}{dE} = \frac{j_c}{nE} \quad (12)$$

It is assumed, for simplicity, that the value of n temperature and magnetic-field independent. Many models have been for the functional form of $j_c(B)$. For the critical current we adopt the power-law model [20], which can be applied over a relatively wide magnetic field range except in the high field region near the upper critical field

$$j_c(B) = j_0(\frac{B_0}{B})^{\gamma} \quad (13)$$

where j_0 and B_0 are the characteristic values of the current density and magnetic field induction; γ is the dimensionless pinning parameter, usually $0 < \gamma < 1$. If we assume $\gamma = 0$, the above model reduces to the Bean-London model [2]. This model is applicable to the case where j_c can be regarded approximately field independent. This power-law model for the critical current has also been used by other groups Another possible decay law would be exponential, which has often been used to take into account its decrease with the magnetic field

$$j(B) = j_0 exp(-\frac{B}{B_0})^{\gamma} \quad (14)$$

where B_0 is a phenomenological parameter related to the pinning ability: the smaller it is, the more drastic is the decrease of the critical current with field. The numerical methods have been applied to resolve the flux diffusion equation, employing the exponential critical state model [14]. Next, based on the power law and exponential models, we shall study the distribution of the magnetic induction, current density and magnetization of superconductors.

3. Basic equations

We formulate the general equation governing the dynamics of the magnetic field induction in a superconductor sample. We study the evolution of the magnetic penetration process in a simple geometry - superconducting semi-infinitive sample $x \geq$. We assume that the external magnetic field induction $\vec{B} = (0, 0, B_e)$ is parallel to the z-axis. When the magnetic field with the flux density $\vec{B} = (0, 0, B_e)$ is applied in the direction of the z-axis, the transport current j(r,

t) and the electric field $\vec{E} = (0, E_e, 0)$ are induced inside the slab along the y-axis. For this geometry, the spatial and temporal evolution of magnetic field induction B (r, t) is described by the following nonlinear diffusion equation in the generalized dimensionless form [7]

$$\frac{db}{dt} = \frac{d}{dt}[b^{\gamma n+1}|\frac{db}{dt}|^{n-1}\frac{db}{dt}] \tag{15}$$

where we have introduced the dimensionless parameters $b = B/B_0, x_p = \mu_0 j_0/B_0, t = t/\tau_0, j = j/j_0, \varepsilon = E/v_0 j_0, B_0 = \mu_0 j_0 v_0 \tau$

It should be noted that the nonlinear diffusion-type equation (15), completed by the flux creep equation (14), totally determine the problem of the space-time distribution of the temperature and electromagnetic field profiles in the flux creep regime with a nonlinear current-voltage characteristics in a semi-infinite superconductor sample. It should be noted that the investigation of the stability conditions in the flux creep regime is very difficult due to absence of solution of equation (15) together with nonlinear j(E) dependence. However, in some limiting cases, it can be solved the problem if we take into account that the heating due to viscous flux motion is negligibly small and get some approximate solution, describing the evolution of thermal and magnetic field diffusion in the creep regime. The diffusion equation (15) can be integrated analytically subject to appropriate initial and boundary conditions in the center of the sample and on the sample's edges. We consider the case, when the magnetic field applied to sample increases with time according to a power law with the exponent of $\alpha > 0$.

$$b(0, t) = b_0 t^{\alpha} \tag{16}$$

Boundary condition (16) is equivalent to a linear increase in the magnetic field with time, which corresponds to a real experimental situation. As can be easily seen that the case $\alpha = 0$ describes a constant applied magnetic field at the surface of the sample, while the case $\alpha = 1$ corresponds to linearly increasing applied field, respectively. The other boundary condition follows from the continuity of the flux at the free boundary $x = x_p$

$$b(x_p, 0) = 0 \tag{17}$$

where x_p is the dimensionless position of the front of the magnetic field. The flux conservation condition for the magnetic field induction can be formulated in the following integral form

$$\int b(x, 0)dx = 1 \tag{18}$$

It should be noted that the nonlinear diffusion equation (15), completed by the boundary conditions for magnetic induction, totally determines the problem of the space-time distribution of the magnetic flux penetration into superconductor sample in the flux flow regime with a power-law dependence of differential resistivity on the magnetic field induction. Solution of this equation gives a complete description of the time and space evolution of the magnetic flux in a sample.

4. Scaling solution In the following analysis we derive an evolution equation for the magnetic induction profile and formulate a similarity solution for the b(x, t). As can be shown that the nonlinear diffusion equation (15) can be solved exactly using well known scaling methods [1, 2]. At long times we present a solution of the nonlinear diffusion equation for the magnetic induction (6) in the following scaling form

$$b(x, t) = t^{\alpha} f(x/t^{\beta}) \tag{19}$$

The similarity exponents α and β are of primary physical importance since the parameter α represents the rate of decay of the magnetic induction b(x, t), while the parameter β is the rate of spread of the space distribution as time goes on. Inserting this scaling form into differential equation (6) and comparing powers of t in all terms, we get the following relationship for the exponents α and β . Using the condition of the flux conservation (18) we obtain

$$\alpha = \beta = 1/(2n + \gamma n + 1) \tag{20}$$

which, suggests the existence of self-similar solutions in the form

$$b(x,t) = t^{1/(2n+\gamma n+1)} f(Z), Z = Xt^{1/(2n+\gamma n+1)} \tag{21}$$

Substituting this scaling solution (16) into the governing equation (15) yields an ordinary differential equation for the scaling function f(z) in the form

$$\frac{d}{dz}[f^{\gamma n+1}|\frac{df}{dz}|^n] + \frac{1}{2n+\gamma n+1}\frac{d}{dz}[z\frac{df}{dz}] = 0 \tag{22}$$

The boundary conditions for the function f(z) now become

$$f(0,t) = 1, f(z_0,t) = 0 \tag{23}$$

The above equation (22), depending on the initial and the boundary conditions describes a scaling—like behavior magnetic flux front with a time—dependent velocity in the sample. After a further integration and applying the boundary conditions (23) we get the following solution of the problem

$$f(z) = f(z_0)[1-(z/z_0)^{(n+1)/n}]^{1/(\gamma+1)} \tag{24}$$

where

$$f(z_0) = [n\tfrac{\gamma+1}{n+1}(\tfrac{z_0^{n+1}}{2n+\gamma n+1})^{1/n}]^{1/(\gamma+1)}$$

The position of the front z_0 can now be found by substituting the solution (24) into the integral condition (18) and it is given by

$$z_0^{(2n+\gamma n+1)/(\gamma+1)} = [\frac{\frac{n}{n+1}F(\frac{\gamma+2}{\gamma+1}+\frac{1}{2})}{\Gamma(\frac{\gamma+2}{\gamma+1})\Gamma(\frac{n}{n+1})}][n\tfrac{\gamma+1}{n+1}(\tfrac{1}{2n+\gamma n+1})^{1/n}]^{1/(\gamma+1)}$$

It is convenient to write the self-similar solution (24) in terms of a primitive variables, as

$$b(x,t) = b_0[1-(\frac{x}{x_p})^{(n+1)/n}]^{1/(\gamma+1)} \tag{25}$$

where

$$b_0(0,t) = b(x,t) = t^{-1/(2n+\gamma n+1)}[n\tfrac{\gamma+1}{n+1}(\tfrac{z_0^{n+1}}{2n+\gamma n+1})^{1/n}]^{1/(\gamma+1)}$$

This solution describes the propagation of the magnetic field into the sample, the magnetic induction being localized in the domain between the surface x=0 and the flux front x_p. This solution is positive in the plane $x_p > x$ and is zero outside of it. Note, that only the x > 0 and t > 0 quarter of the plane is presented, because of it has physical relevance. The penetrating flux front position $x = x_p(t)$ as a function of time can be described by the relation

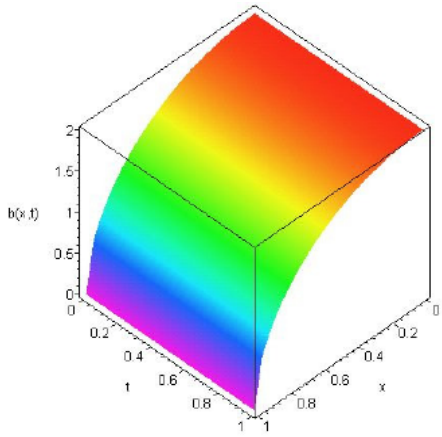
$$x_p = x_0 t^{-1/(2n+\gamma n+1)}$$

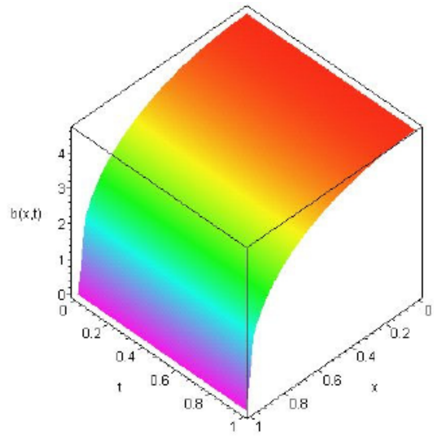
Using the last relation the velocity of the magnetic flux induction can be obtained as the following

$$v_p \approx v_0 t^{-n(2+\gamma)/(2n+\gamma n+1)} \tag{26}$$

Let us now consider the most interesting case n=1. In this particular case the spatial and temporal evolution of the magnetic flux induction is totally determined by the parameters γ, α and β. As the following analysis we may derive an evolution equation for the magnetic induction profile for the case n=1 and apply the scalings of the previous section to formulate a similarity solution for the b(x, t). The time and space evolution of the self-simulating solution of magnetic field penetration into a superconductor for n=1 is shown schematically in figure 1a and 1b.

Fig.1a and 1b. The distributions of the normalized flux density b(x, t) at time t=1, for γ=1, 2.





Conclusion The spatial and temporal profiles of magnetic flux penetration in the sample depends on the set of three independent parameters, n, q and α. It is of interest to consider the nonlinear diffusion equation for the magnetic induction for different values of the exponents n, q and α. For a given parameter set n, q and α the form of the scaling function f(z) has been obtained by solving the nonlinear diffusion equation analytically by a self-similar technique. We have solved the nonlinear diffusion equation analytically to provide expressions for the time-space evolution of the magnetic induction for different values of exponents n, q and α. Next, we systematically analyzed the effect of different values of exponents on the shape of the magnetic flux front in the sample. Varying the parameters of the equation, we may observe a various shapes of the magnetic flux front in the sample [21-26]. A similar approach has been presented in Ref. [7] within the framework of non-linear flux diffusion in transverse geometry. As can be shown below that different values exponents n and q generate different space–time magnetic flux fronts.

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